

Temperature Dependence of Non-Ideal MHD Transport Coefficients: An Isochoric Saha-Equilibrium Benchmark

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Abstract

We compare Ohmic, Hall, and ambipolar transport in an idealized, pure-hydrogen plasma along a prescribed isochoric sequence from 4000 to 20 000 K. The hydrogen ionization fraction is obtained from the closed-form Saha solution. Ohmic diffusivity includes both electron–ion and electron–neutral collisions, with a Coulomb logarithm evaluated at each temperature. Neutral collision rates retain their thermal-speed dependence. For $n_{\text{H}} = 10^{17} \text{ m}^{-3}$ and $B = 50 \text{ G}$, the electron–neutral and electron–ion resistive contributions are comparable at 4000 K; their ratio falls to 0.029 at 5000 K. The ambipolar coefficient exceeds the Hall coefficient below approximately 8990 K and exceeds the Ohmic coefficient below approximately 12 690 K. Hall transport remains non-dissipative, so these crossings characterize coefficient ordering rather than competing heating rates. A nine-case density and magnetic-field survey quantifies the parameter dependence. Analytic limits explain why the ambipolar-to-Hall ratio loses its leading dependence on ionization fraction at low ionization, and on density near complete ionization. A matched-state comparison with published VAL-C electron densities quantifies the limitations: the Saha-based Hall coefficient is about 35 times larger at the temperature minimum, but differs by only 4.1% at 18 500 K. The calculation is an LTE benchmark, not a time-dependent model of rapid chromospheric heating or a prediction for an observed atmosphere.

Keywords: partially ionized plasma; magnetohydrodynamics; Saha equilibrium; Ohmic diffusion; Hall effect; ambipolar diffusion.

1 Introduction

The electrical response of a partially ionized plasma depends on its charged-particle abundance and collisional coupling. Comparing transport coefficients can help identify relevant terms in the induction equation, provided their geometry, characteristic scales, and distinct energetic roles are respected.

Khomenko and Collados [1] already compared Ohmic, Hall, and ambipolar coefficients in a stratified magnetic atmosphere using a VAL-C temperature structure. Pandey and Wardle [2] developed a single-fluid description connecting weakly and strongly ionized regimes. The present work does not introduce new transport physics. Its contribution is a reproducible temperature-controlled benchmark: explicit equilibrium ionization, specified collision prescriptions, numerical coefficient crossings, and analytic density and field scalings. A separate comparison at published VAL-C thermodynamic states quantifies the error introduced by replacing its electron-density closure with pure-hydrogen Saha equilibrium.

We deliberately separate this benchmark from rapid heating. Time-dependent hydrogen ionization in the solar atmosphere can depart strongly from LTE [3]. Consequently, the temperature coordinate below labels independently prescribed equilibrium states, not successive times during reconnection.

2 Model and methods

2.1 Composition and thermodynamic path

The gas contains ground-state neutral hydrogen, protons, and electrons, with a common Maxwellian temperature. Define

$$n_{\text{H}} = n_{\text{n}} + n_{\text{i}}, \quad x_{\text{e}} = \frac{n_{\text{e}}}{n_{\text{H}}} = \frac{n_{\text{i}}}{n_{\text{H}}}, \quad \xi_{\text{n}} = 1 - x_{\text{e}}. \quad (1)$$

Charge neutrality is imposed. Approximating the neutral and ion masses by m_{H} and neglecting electron mass in the bulk density,

$$\rho = m_{\text{H}}n_{\text{H}}, \quad \rho_{\text{i}} = x_{\text{e}}\rho, \quad \rho_{\text{n}} = \xi_{\text{n}}\rho, \quad \rho_{\text{e}} = m_{\text{e}}x_{\text{e}}n_{\text{H}}. \quad (2)$$

The reference sequence has $n_{\text{H}} = 10^{17} \text{ m}^{-3}$, $B = 50 \text{ G} = 5 \times 10^{-3} \text{ T}$, and $4000 \leq T \leq 20\,000 \text{ K}$. Both density and field magnitude are held fixed for coefficient evaluation. Thus the path is isochoric, with $p = (1 + x_{\text{e}})n_{\text{H}}k_{\text{B}}T$ changing along it; neither pressure balance nor an energy equation is imposed. These independently selected parameters are not a VAL-C atmosphere. VAL-C is a height-dependent atmospheric model [4], rather than a prescription for a fixed-density temperature sweep or for B . Additional sequences use every combination of $n_{\text{H}} = 10^{16}, 10^{17}, 10^{18} \text{ m}^{-3}$ and $B = 10, 50, 100 \text{ G}$.

2.2 Saha equation and stable solution

The hydrogen Saha relation [5] is

$$\frac{n_{\text{e}}n_{\text{i}}}{n_{\text{n}}} = S(T) = \frac{2U_{\text{i}}}{U_{\text{n}}} \left(\frac{2\pi m_{\text{e}}k_{\text{B}}T}{h^2} \right)^{3/2} \exp\left(-\frac{\chi_{\text{H}}}{k_{\text{B}}T}\right), \quad (3)$$

where $\chi_{\text{H}} = 13.5984346 \text{ eV}$. We adopt ground-state partition functions $U_{\text{n}} = 2$ and $U_{\text{i}} = 1$; the explicit factor 2 accounts for electron spin. Hence $2U_{\text{i}}/U_{\text{n}} = 1$. Excited-state partition-function corrections are outside this benchmark.

Writing $q = S(T)/n_{\text{H}}$, the quadratic and its physical root are

$$x_{\text{e}}^2 + qx_{\text{e}} - q = 0, \quad x_{\text{e}} = \frac{2}{1 + \sqrt{1 + 4/q}}, \quad \xi_{\text{n}} = \frac{x_{\text{e}}^2}{q}. \quad (4)$$

The rationalized root avoids subtracting nearly equal numbers; the last expression also avoids cancellation in $1 - x_{\text{e}}$ near complete ionization. Ionization is therefore solved analytically and the resulting coefficients are evaluated numerically.

2.3 Collision prescription and Coulomb logarithm

For neutral collisions we use the explicit approximate prescription of Khomenko and Collados [1]:

$$\begin{aligned} K_{j\text{n}}(T) &= \sigma_{j\text{n}} \sqrt{\frac{8k_{\text{B}}T}{\pi m_{j\text{n}}}}, & \nu_{j\text{n}} &= n_{\text{n}}K_{j\text{n}}, \\ m_{j\text{n}} &= \frac{m_j m_{\text{H}}}{m_j + m_{\text{H}}}, & \sigma_{\text{en}} &= 10^{-19} \text{ m}^2, \quad \sigma_{\text{in}} = 5 \times 10^{-19} \text{ m}^2, \end{aligned} \quad (5)$$

where $j = \text{e, i}$. Each K has units $\text{m}^3 \text{ s}^{-1}$ and each ν has units s^{-1} . The cross sections are fixed effective inputs; they are not energy-resolved scattering calculations.

The electron–ion frequency is

$$\nu_{\text{ei}} = \frac{n_{\text{e}}e^4 \ln \Lambda}{3m_{\text{e}}^2 \epsilon_0^2} \left(\frac{m_{\text{e}}}{2\pi k_{\text{B}}T} \right)^{3/2}. \quad (6)$$

For the Coulomb logarithm we use the low-temperature electron-ion branch of the NRL Plasma Formulary [6]:

$$\ln \Lambda = 23 - \ln \left(N_e^{1/2} \Theta^{-3/2} \right), \quad N_e = \frac{n_e}{10^6 \text{ m}^{-3}}, \quad \Theta = \frac{k_B T}{1 \text{ eV}}. \quad (7)$$

Both arguments are dimensionless: N_e is the numerical density in cm^{-3} and Θ the electron temperature in eV. The applicable branch has $T_i m_e / m_i < T_e < 10$ eV for singly charged ions; equal species temperatures satisfy it throughout this sweep. No fixed coronal Coulomb logarithm is used.

2.4 Transport coefficients and drag convention

We adopt the leading single-fluid collision closure

$$\sigma_{\parallel} = \frac{n_e e^2}{m_e (\nu_{ei} + \nu_{en})}, \quad \eta_O = \frac{1}{\mu_0 \sigma_{\parallel}} = \eta_{O,ei} + \eta_{O,en}, \quad (8)$$

$$\eta_H = \frac{B}{\mu_0 e n_e}, \quad \eta_A = \frac{\xi_n^2 B^2}{\mu_0 \alpha_n}, \quad (9)$$

$$\alpha_n = \rho_i \nu_{in} + \rho_e \nu_{en}. \quad (10)$$

All three η coefficients have units $\text{m}^2 \text{s}^{-1}$; B denotes the nonnegative field magnitude. These expressions specify a fluid approximation, not a full kinetic conductivity tensor. Its Coulomb contribution has the classical Spitzer temperature scaling, without assuming Coulomb dominance at every temperature.

To remove ambiguity in the drag normalization, define

$$C(T) = m_H K_{in} + m_e K_{en}, \quad \gamma_{\text{eff}} = \frac{C(T)}{m_H^2}, \quad \alpha_n = \gamma_{\text{eff}} \rho_i \rho_n. \quad (11)$$

Thus $[\alpha_n] = \text{kg m}^{-3} \text{s}^{-1}$ and $[\gamma_{\text{eff}}] = \text{m}^3 \text{kg}^{-1} \text{s}^{-1}$. The ion contribution alone is $\gamma_{in} = K_{in}/m_H$ in the frequency convention of Eq. (5). A drag coefficient defined using another collision-rate convention cannot be inserted without conversion. In this notation,

$$\eta_A = \frac{B^2}{\mu_0 \gamma_{\text{eff}} \rho^2} \frac{\xi_n}{x_e}. \quad (12)$$

The μ_0 factor is required in SI units. The fully ionized limit is evaluated continuously, giving $\eta_A \rightarrow 0$.

2.5 Geometry, energetics, and dimensionless comparisons

Neglecting electron inertia and pressure-gradient battery terms, the retained generalized Ohm law is

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mu_0 \left(\eta_O \mathbf{J} + \eta_H \mathbf{J} \times \hat{\mathbf{b}} + \eta_A \mathbf{J}_{\perp} \right), \quad (13)$$

where $\mathbf{v}$ is the bulk velocity, $\hat{\mathbf{b}} = \mathbf{B}/B$, $\mathbf{J} = \nabla \times \mathbf{B}/\mu_0$, and $\mathbf{J}_{\perp} = \mathbf{J} - (\mathbf{J} \cdot \hat{\mathbf{b}})\hat{\mathbf{b}}$. The irreversible heating rate is

$$Q = \mu_0 \eta_O J^2 + \mu_0 \eta_A J_{\perp}^2, \quad \frac{Q_A}{Q_O} = \frac{\eta_A}{\eta_O} \sin^2 \theta, \quad (14)$$

with θ the angle between current and magnetic field. Hall transport contributes no direct heating because $\mathbf{J} \cdot (\mathbf{J} \times \hat{\mathbf{b}}) = 0$.

We report $R_{HO} = \eta_H/\eta_O$, $R_{AO} = \eta_A/\eta_O$, and $R_{AH} = \eta_A/\eta_H$. For a common characteristic length L , velocity U , and comparable nonvanishing geometric factors, define

$$\mathcal{D}_j = \frac{\eta_j}{UL}, \quad t_j = \frac{L^2}{\eta_j}, \quad j = O, H, A. \quad (15)$$

The $\mathcal{D}_j$ estimate non-ideal terms relative to induction by bulk motion. For Hall transport, t_H is a field evolution scale, not a dissipation time. Coefficient ratios alone do not establish dynamical dominance or a heating rate.

2.6 Evaluation and numerical checks

We use $e = 1.602176634 \times 10^{-19}$ C, $k_B = 1.380649 \times 10^{-23}$ J K⁻¹, $h = 6.62607015 \times 10^{-34}$ J s, $m_e = 9.1093837139 \times 10^{-31}$ kg, $\epsilon_0 = 8.8541878188 \times 10^{-12}$ F m⁻¹, and $\mu_0 = 1.25663706127 \times 10^{-6}$ H m⁻¹ [7]. The approximate common hydrogen mass is $m_H = 1.6735575 \times 10^{-27}$ kg.

Equations (4)–(11) were evaluated in double precision for all nine parameter combinations. Crossings were bracketed on a 10 K grid and refined by a scalar root solver. Checks on a 1 K grid verified positivity, the Saha quadratic, $x_e + \xi_n = 1$, and the ratio identities below. Quoted crossover temperatures are rounded to 10 K to avoid implying physical accuracy beyond the collision model.

For an independent closure comparison, we also take six (T, n_H, n_e) entries directly from Table 12 (p. 680) of Vernazza et al. [4], converting cm⁻³ to m⁻³. At each tabulated (T, n_H) we recalculate the pure-hydrogen Saha electron density. These additional states belong to a stratified atmosphere; their densities are not restricted to the nine-case isochoric survey. We compare the leading Hall coefficient at a common field strength, so that its ratio depends only on the two electron densities. The published values include non-LTE and composition effects; this comparison does not isolate either effect separately.

3 Analytic limits

The two Ohmic contributions scale as

$$\eta_{O,ei} \propto (\ln \Lambda) T^{-3/2}, \quad \eta_{O,en} = \frac{m_e K_{en}}{\mu_0 e^2} \frac{\xi_n}{x_e} \propto T^{1/2} \frac{\xi_n}{x_e}. \quad (16)$$

Only the second term retains the steep explicit ionization dependence. Their sum cannot generally be replaced by a single $T^{-3/2}$ law.

Cancellation of common factors gives

$$R_{HO} = \frac{eB}{m_e(\nu_{ei} + \nu_{en})}, \quad R_{AH} = \frac{eB\xi_n}{n_H C(T)}. \quad (17)$$

At fixed T and density, η_O is independent of B , $\eta_H \propto B$, and $\eta_A \propto B^2$. For $x_e \ll 1$, Eq. (17) becomes

$$R_{AH} \simeq \frac{eB}{n_H C(T)} \propto \frac{B}{n_H T^{1/2}}. \quad (18)$$

Thus the leading $1/x_e$ dependence cancels between ambipolar and Hall transport. Their individual coefficients can change strongly while their ratio changes much less.

In the opposite limit, $\xi_n \simeq n_H/S(T)$, so

$$R_{AH} \simeq \frac{eB}{C(T)S(T)}, \quad \eta_A \simeq \frac{B^2}{\mu_0 n_H C(T)S(T)}. \quad (19)$$

The leading density dependence cancels from R_{AH} near complete ionization. Since $C \propto T^{1/2}$, the Hall–ambipolar crossing approximately satisfies

$$T^2 \exp\left(-\frac{\chi_H}{k_B T}\right) \propto B. \quad (20)$$

This explains the weak density dependence of that crossing in the numerical survey; it is a property of this equilibrium model.

4 Results

4.1 Ionization and resistive contributions

At the reference density, x_e increases from 2.12×10^{-4} at 4000 K to 0.186 at 6000 K and 0.980 at 8000 K. It reaches 0.5 at approximately 6580 K and 0.99 at approximately 8290 K. At 20 000 K the neutral fraction is 3.91×10^{-8} , rather than exactly zero.

Table 1: Reference sequence for $n_{\text{H}} = 10^{17} \text{ m}^{-3}$ and $B = 50 \text{ G}$. All transport coefficients are in $\text{m}^2 \text{ s}^{-1}$. The neutral fraction is listed to retain information near full ionization. Transport coefficients are rounded to three significant figures.

$T \text{ (K)}$	x_{e}	ξ_{n}	η_{O}	η_{H}	η_{A}
4000	2.12×10^{-4}	0.999788	1.05×10^4	1.17×10^9	8.62×10^{11}
5000	0.0129	0.987	3.36×10^3	1.93×10^7	1.25×10^{10}
6000	0.186	0.814	2.25×10^3	1.34×10^6	6.54×10^8
8000	0.980	0.0205	1.40×10^3	2.54×10^5	2.70×10^6
10 000	0.999705	2.95×10^{-4}	1.04×10^3	2.48×10^5	3.41×10^4
15 000	0.9999992	8.35×10^{-7}	5.99×10^2	2.48×10^5	7.89×10^1
20 000	0.99999996	3.91×10^{-8}	4.05×10^2	2.48×10^5	3.20

Table 2: Crossings for the nine equilibrium sequences. T_{AH} and T_{AO} denote coefficient equalities, not universal dynamical transitions. No Hall–Ohmic crossing occurs between 4000 and 20 000 K.

$n_{\text{H}} \text{ (m}^{-3}\text{)}$	$B \text{ (G)}$	$T_{\text{AH}} \text{ (K)}$	$T_{\text{AO}} \text{ (K)}$
10^{16}	10	8300	11 760
10^{16}	50	8990	15 240
10^{16}	100	9320	17 430
10^{17}	10	8300	10 170
10^{17}	50	8990	12 690
10^{17}	100	9320	14 180
10^{18}	10	8220	8950
10^{18}	50	8970	10 860
10^{18}	100	9310	11 950

The calculated Coulomb logarithm spans 9.71–12.97. Fixing it at 20 would overestimate the Coulomb resistive contribution by factors of approximately 1.54–2.06 over this sequence. At 4000 K,

$$\nu_{\text{en}} = 3.93 \times 10^3 \text{ s}^{-1}, \quad \nu_{\text{ei}} = 3.95 \times 10^3 \text{ s}^{-1}, \quad \eta_{\text{O}} = 1.05 \times 10^4 \text{ m}^2 \text{ s}^{-1}. \quad (21)$$

Omitting neutral collisions would give only $5.26 \times 10^3 \text{ m}^2 \text{ s}^{-1}$ there. The ratio $\eta_{\text{O,en}}/\eta_{\text{O,ei}}$ is 0.995, 0.0292, and 0.00265 at 4000, 5000, and 6000 K, respectively. With these particular cross sections, their equality lies near 4000 K, just below the sampled interval. Electron–neutral collisions therefore matter near the cool boundary but do not dominate most of this reference sequence. At $n_{\text{H}} = 10^{18} \text{ m}^{-3}$, equality moves to approximately 4290 K.

Across 4000–20 000 K, γ_{eff} increases from 3.89×10^{12} to $8.69 \times 10^{12} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-1}$, exactly a factor $\sqrt{5}$ for the adopted fixed cross sections. Meanwhile $\xi_{\text{n}}/x_{\text{e}}$ decreases by about 1.21×10^{11} . Ionization therefore controls the much larger ambipolar variation, while the thermal dependence of drag is retained explicitly.

4.2 Coefficient ordering and field–density survey

Table 1 gives absolute values, and Fig. 1 shows ionization, coefficients, and ratios. In the reference case,

$$\eta_{\text{A}} = \eta_{\text{H}} \quad \text{at} \quad T_{\text{AH}} \simeq 8990 \text{ K}, \quad \eta_{\text{A}} = \eta_{\text{O}} \quad \text{at} \quad T_{\text{AO}} \simeq 12 690 \text{ K}. \quad (22)$$

Thus $\eta_{\text{A}} > \eta_{\text{H}} > \eta_{\text{O}}$ below T_{AH} , $\eta_{\text{H}} > \eta_{\text{A}} > \eta_{\text{O}}$ between the crossings, and $\eta_{\text{H}} > \eta_{\text{O}} > \eta_{\text{A}}$ above T_{AO} . There is no Hall–Ohmic crossing: the minimum R_{HO} is approximately 175 near 7540 K.

For example, at 8000 K, $R_{\text{AH}} = 10.7$, $R_{\text{HO}} = 181$, and $R_{\text{AO}} = 1.92 \times 10^3$. At 10 000 K these become 0.137, 239, and 32.9. The latter case illustrates why a large Hall coefficient does not remove ambipolar heating of perpendicular currents.

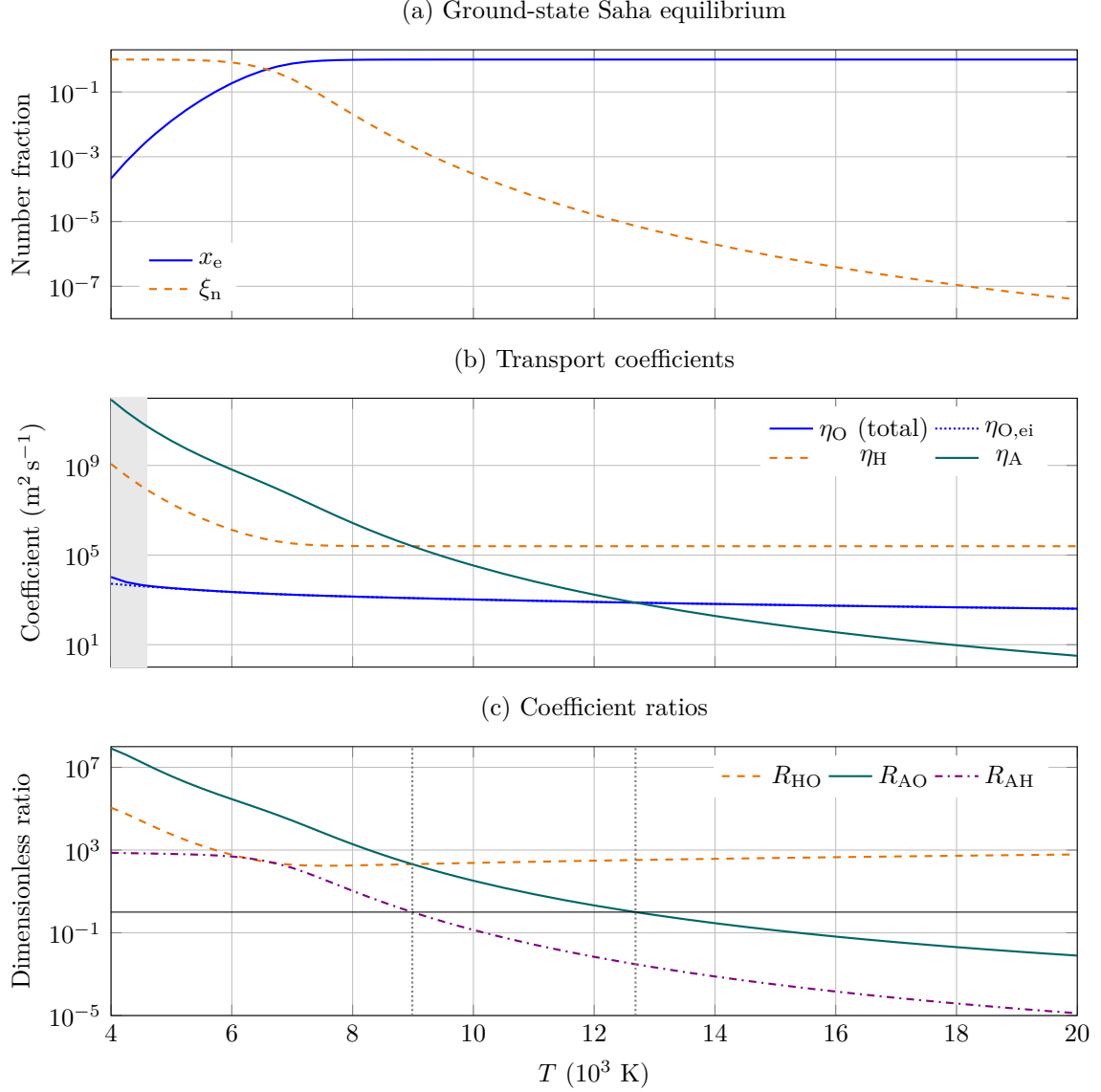


Figure 1: Reference LTE sequence at $n_{\text{H}} = 10^{17} \text{ m}^{-3}$ and $B = 50 \text{ G}$. Vertical axes are logarithmic. In panel (b), shading marks $\eta_{\text{O,en}}/\eta_{\text{O,ei}} > 0.1$; outside it the Coulomb-only curve is a better approximation to the specified collision closure. In panel (c), vertical lines mark the ambipolar–Hall and ambipolar–Ohmic crossings. All panels assume pure hydrogen in LTE. The shading does not define a real chromospheric LTE boundary; dynamical application additionally requires the coupling and scale checks in Section 5.1.

As Table 2 shows, increasing B raises both crossing temperatures. At 50 G, a hundredfold density change moves T_{AH} by only about 20 K, consistently with Eq. (19). The ambipolar–Ohmic crossing varies much more, because the density cancellation does not apply to that ratio. Across all nine cases, the smallest R_{HO} is approximately 4.57.

For an illustrative scale comparison at 8000 K, take $U = 10 \text{ km s}^{-1}$ and $L = 100 \text{ km}$. Then

$$(\mathcal{D}_{\text{O}}, \mathcal{D}_{\text{H}}, \mathcal{D}_{\text{A}}) = (1.40 \times 10^{-6}, 2.54 \times 10^{-4}, 2.70 \times 10^{-3}). \quad (23)$$

All are small relative to the bulk induction estimate, although ambipolar transport is the largest non-ideal coefficient. These scales are illustrative inputs, not measured properties.

5 Discussion and limitations

5.1 Magnetization and single-fluid validity

Define

$$\beta_{\text{e}} = \frac{eB/m_{\text{e}}}{\nu_{\text{ei}} + \nu_{\text{en}}}, \quad \beta_{\text{i}} = \frac{eB/m_{\text{H}}}{\nu_{\text{in}} + \nu_{\text{ie}}}, \quad \nu_{\text{ie}} = \frac{m_{\text{e}}}{m_{\text{H}}} \nu_{\text{ei}}. \quad (24)$$

Along the reference sequence, β_{e} ranges from approximately 175 to 1.12×10^5 , and β_{i} from 170 to 736. Both species are magnetized according to these collisional measures. The familiar weak-ionization Hall-dominated ordering $\beta_{\text{i}} \ll 1 \ll \beta_{\text{e}}$ is therefore not assumed. Retaining the Hall term in a generalized Ohm law does not require that ordering; its importance also depends on neutral fraction and dynamical scale [2].

The electron contribution to neutral drag satisfies $\rho_{\text{e}}\nu_{\text{en}}/(\rho_{\text{i}}\nu_{\text{in}}) = 0.00330$ for this prescription. It is retained in α_{n} , while the Hall coefficient uses the leading single-fluid form. Higher-order drift corrections and a full kinetic resistivity tensor are not evaluated.

A single-fluid application additionally requires small relative drifts and sufficiently slow evolution. As a conservative neutral-coupling diagnostic where neutrals carry appreciable mass, compare the relevant evolution rates with

$$\nu_{\text{ni}} = \frac{\rho_{\text{i}}}{\rho_{\text{n}}} \nu_{\text{in}} = x_{\text{e}} n_{\text{H}} K_{\text{in}}. \quad (25)$$

At 4000 K, $\nu_{\text{ni}} = 0.137 \text{ s}^{-1}$. Requiring the estimated ambipolar evolution rate to satisfy $\eta_{\text{A}}/L^2 \ll \nu_{\text{ni}}$ gives $L \gg 2.50 \times 10^6 \text{ m}$ at that endpoint. Thus the very large low-temperature coefficient cannot consistently be used to infer an arbitrarily short diffusion time at small scales. This conservative diagnostic is not a substitute for the full frequency and drift conditions of a chosen dynamical model. The example at 8000 K has $\nu_{\text{ni}} \simeq 898 \text{ s}^{-1}$ and passes this particular coupling check comfortably.

5.2 LTE, composition, and collision uncertainty

Saha equilibrium requires thermodynamic conditions that are not guaranteed merely by waiting for a collision time. Non-LTE radiation and finite ionization/recombination rates can invalidate it in the chromosphere. Simulations show that hydrogen ionization can retain memory of earlier hot phases [3]. Accordingly, these curves are neither a validated rapid-heating trajectory nor a general upper or lower bound on real electron density. A time-dependent application requires rate equations and an appropriate radiation field.

The pure-hydrogen approximation also restricts interpretation. At 4000 K, the predicted electron abundance is only 2.12×10^{-4} per hydrogen nucleus; an omitted donor population of comparable electron abundance would alter the Hall coefficient and the resistive balance substantially. Metals, helium, molecular species, and their contributions to mass and collisions are not included. No solar-composition electron floor is imposed, and no interstellar application is inferred.

Real scattering data are more detailed than fixed effective cross sections; quantitative atmospheric calculations require appropriate collision inputs [8]. As a sensitivity experiment, multiplying σ_{in} by 0.5 or 2 changes the reference T_{AH} to approximately 9320 or 8680 K, and T_{AO} to 13390 or 12050 K. Multiplying σ_{en} by 0.5 or 2 changes $\nu_{\text{en}}/\nu_{\text{ei}}$ at 4000 K to 0.498 or 1.99. These are controlled

input variations, not confidence intervals. They show why the crossings are conditional model results rather than precise atmospheric temperatures.

5.3 Quantitative comparison with stratified atmosphere inputs

Khomenko and Collados [1] already obtained large ambipolar-to-Ohmic ratios in a stratified magnetic atmosphere. The local coefficient forms used here follow their Eqs. (18)–(20), with the collision and Coulomb-logarithm choices stated explicitly above. Agreement of these algebraic forms is not an independent validation of the Saha closure. Their height-dependent field and density also prevent direct identification of their coefficient crossings with temperatures on our fixed-density sequence.

To test a specific consequence of the simplified closure, Table 3 compares electron densities at identical local (T, n_{H}) states taken from the published VAL-C table [4]. Denote the tabulated electron density by $n_{\text{e}}^{\text{VAL}}$ and the recalculated value by $n_{\text{e}}^{\text{S}} = x_{\text{e}}(T, n_{\text{H}})n_{\text{H}}$. Applying Eq. (9) to each density at the same B gives

$$F_{\text{H}} \equiv \frac{\eta_{\text{H}}^{\text{S}}}{\eta_{\text{H}}^{\text{VAL}}} = \frac{n_{\text{e}}^{\text{VAL}}}{n_{\text{e}}^{\text{S}}}. \quad (26)$$

This ratio is independent of the illustrative magnetic field; no magnetic field is attributed to VAL-C itself.

Table 3: Matched-state electron-density and Hall-coefficient comparison. Height z , temperature, hydrogen density, and $n_{\text{e}}^{\text{VAL}}$ are from Table 12 of Vernazza et al. [4]; the last two columns are calculated here. All number densities are in m^{-3} . $F_{\text{H}} > 1$ means that the pure-hydrogen Saha closure overestimates the leading Hall coefficient. These are selected local atmospheric states, not points on the fixed-density reference sequence.

z (km)	T (K)	n_{H}	$n_{\text{e}}^{\text{VAL}}$	n_{e}^{S}	F_{H}
515	4170	2.096×10^{21}	2.495×10^{17}	7.077×10^{15}	35.3
1065	6040	1.711×10^{19}	9.349×10^{16}	2.930×10^{17}	0.319
1605	6440	6.386×10^{17}	6.005×10^{16}	1.214×10^{17}	0.495
1990	7160	1.033×10^{17}	3.858×10^{16}	8.492×10^{16}	0.454
2104	9500	5.239×10^{16}	3.705×10^{16}	5.237×10^{16}	0.707
2113	18 500	2.732×10^{16}	2.620×10^{16}	2.732×10^{16}	0.959

At the temperature minimum, the Saha electron density is lower by a factor of 35.3, causing the same factor of overestimation in the Hall coefficient. At 6040–7160 K the discrepancy changes sign: the Saha Hall coefficient is only 0.319–0.495 of the value obtained from the tabulated electron density. At 18 500 K the Hall coefficients agree to 4.1%, consistent with approaching almost complete hydrogen ionization. Agreement of electron densities near this limit does not establish agreement of ambipolar coefficients, which remain sensitive to the small residual neutral abundance.

This comparison demonstrates quantitatively that the Saha sequence is neither a general upper nor a lower bound on chromospheric electron density. It tests the ionization and composition closure through a common Hall expression, rather than reproducing the full magnetic stratification or the Ohmic and ambipolar profiles of Khomenko and Collados. A complete atmospheric transport comparison additionally requires consistent species-resolved densities, collision inputs, and magnetic geometry. The contribution of the present model is therefore a controlled analytic and numerical reference with quantified closure limitations, not a new atmospheric regime map.

6 Conclusion

Including electron–neutral collisions removes the assumption of Coulomb-only resistivity at the cool boundary of a Saha-equilibrium hydrogen sequence. At 10^{17} m^{-3} and 50 G, neutral and Coulomb resistive contributions are comparable at 4000 K, while Coulomb collisions dominate above that boundary for the adopted cross sections. A temperature-dependent Coulomb logarithm and an

explicitly normalized drag coefficient yield ambipolar–Hall and ambipolar–Ohmic crossings near 8990 and 12 690 K, respectively.

The density–field survey and limiting expressions explain how these crossings move. Hall transport must be interpreted through induction and scale comparisons, while dissipation depends on Ohmic and ambipolar coefficients and current orientation. The matched-state VAL-C comparison shows that replacing a non-LTE, multicomponent electron density by pure-hydrogen Saha equilibrium can change the Hall coefficient by a factor of 35 near the temperature minimum and can reverse the sign of the error elsewhere. The resulting reference is useful within its stated pure-hydrogen, LTE, and collision-closure assumptions; application to a real chromosphere requires ionization kinetics and radiation, composition, geometry, and dynamical coupling to be evaluated separately.

Reproducibility

No new observational data are used. The six published atmospheric states used in Table 3 are identified above. All constants, equilibrium relations, collision inputs, and parameter combinations are specified. The accompanying Python script, `diffusion_calculations.py`, evaluates the model, locates the crossings, verifies the stated numerical identities, and exports the figure data and tabulated results. Figure 1 is drawn by PGFPlots from numerical samples embedded in this LaTeX source, which compiles without external images or data files. Python is needed only to regenerate the numerical samples.

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